An improved LALR(\(k\)) parser generation for regular right part grammars

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1. Introduction

A regular right part grammar (RRPG) is a context-free grammar, in which right parts of productions are finite automata to extend the descriptive power of context-free grammar by including notations for describing repetitions and alternations [6,8]. On LR parsing of RRPGs, extra work is required to identify the left end of a handle at reduction time because a nonterminal can derive potentially infinite number of strings via a single production.

For parsing RRPGs, some methods such as grammar transformation from RRPGs to LR(\(k\)) context-free grammars [5,8], augmenting LR(0) automaton with readback machines to recognize the reverse of state sequences leading to a reduction [2,6,7], and stacking an LR state only when the symbol being processed indicates the beginning of a new right part [11], are suggested. In stacking method, the parser is efficient because exactly one state entry is popped from the stack when the right end of a production is found. However, if stacking conflicts occur, the transformations to another RRPG whose parser has no stacking conflict are necessary [11].

Another method is the combination of stacking method and readback method [9]. When stacking conflicts occur while reading the right part of a production, several state entries are popped until one of the lookback states (in which parser may be restarted after reduction [2]) for a reduction appears. Readback states which are used to construct readback machine need not be added. Moreover, stacking conflicts are resolved by using lookback states without grammar transformation. Both the parser and the generation of the parsers are efficient [9]. In the method, the right parts of productions are deterministic finite automata.

Our approach is based on the method of Nakata and Sassa [9]. Even though their method is very simple and can produce small and fast parsers if there is no stacking conflict, it requires a very large amount of space for realistic grammars in keeping the relations on (state, item) pairs [9]. To reduce space requirements, we use only kernel items of parser states on building the parser. An improved parser generation algorithm for RRPGs which allows the right parts of productions to be nondeterministic finite automata, is presented. It is well known that for succinct-
ness of classes of language descriptions, nondeterministic finite automata can be exponentially more succinct than deterministic finite automata [12].

2. Terminology and basic definitions

The reader is referred to [1,12] for notations and definitions relating to relations, strings, languages, and automata.

A nondeterministic finite automaton $M_0 = (Q_0, V, δ_0, q_0, F_0)$ where $Q_0$ is a finite set of states, $V$ is a finite set of transition symbols, $δ_0: Q_0 × V → 2^{Q_0}$ is the state transition function, $q_0 ∈ Q_0$ is the initial state, and $F_0 ⊆ Q_0$ is the set of final states. $δ_0$ is extended to a mapping $δ_0: Q_0 × V^∗ → 2^{Q_0}$ as follows:

$$δ_0(q, ε) = \{q\},$$
$$δ_0(q, αX) = \{p | ∃ r ∈ δ_0(q, α): p ∈ δ_0(r, X)\}$$

where $Z ∈ V$, $α ∈ V^∗$.

A regular right part grammar $G = (N, Σ, P, S)$ where $N$ and $Σ$ are finite set of nonterminal symbols and terminal symbols such that $N ∩ Σ = ∅$, respectively, $P ⊆ N × M_N$ is a finite set of productions of the form $A → M_A$ where $A$, the left part, is in $N$ and $M_A$, the right part, is a nondeterministic finite automaton recognizing a subset of $V^∗$ where $M_A = (Q_A, V, δ_A, q_A, F_A)$ and $V = N ∪ Σ$, and $S ∈ N$ is the start symbol; in which

$$M_N = \bigcup_{A ∈ N} \{M_A\}, \quad Q = \bigcup_{A ∈ N} Q_A,$$
$$A_I = \bigcup_{A ∈ N} \{q_A\}, \quad Q_F = \bigcup_{A ∈ N} F_A,$$
$$|N| = |M_N| = |Q_I|,$$

and $δ$ is the collection of $δ_A$ for all $A$ in $N$. For each production $A → M_A$, we write it more conveniently as $A → q_A$ or simply $A → q$ if no ambiguity can arise, and the sets $Q_A$ are assumed to be disjoint as original definition of standard form defined in [6]. The definitions of useless symbol and reduced grammar can be adapted from context-free grammars in a straightforward way. It will be assumed that RRPGs are reduced and in $S$-augmented form [2]. Figure 1 shows an RRPG $G_1$ whose productions are represented by transition graphs. Using regular expressions, these productions might be written as

$$S' → S, \quad S → (a|ab)b^*c|aA,$$
$$A → b(a|Bc), \quad B → cb.$$

The formalism used in [4] to describe LR parsers for context-free grammars can be adapted for RRPGs with slight modifications.

An extended LR(0) (ELR(0)) automaton for an RRPG $G$, $ELR_0(G) = (I, V, P, I_0, Next, Reduce)$ where $I$ is a finite set of states, $V$ and $P$ are as in $G$, $I_0 ∈ I$ is the start state, $Next: I × V → I$ is the transition function which may be a partial function, and $Reduce: I × Σ → 2^P$ is the reduce function. To avoid confusion an ELR(0) automaton state is referred to as a state and a state of production right part is referred to as a right part state. A state $I_q$ is inconsistent iff there exists an $a ∈ Σ$ such that $Next(I_q, a) ∈ I$ and $Reduce(I_q, a) ≠ ∅$, $|Reduce(I_q, a)| > 1,$ or both.

The classical construction for building LR(0) automata [3] can be applied to ELR(0) automata by defining an item to be a dotted right part state which is of the form $[A → q:p]$ or simply $[p]$ where $A → q$ is a production and $p$ is its right part state, and identifying ELR(0) states with sets of items. Each state $I_q$ consists of kernel items and nonkernel items which are denoted as Ker-
nel(Iq) and Nonkernel(Iq). In particular, initial
state I0 has only nonkernel items [9].

An operator δ is a mapping in the power set of
items as used in [10], which is defined as
\[ δ([p]) = \{ [q] \mid \exists A ∈ N: δ(p, A) ≠ ∅ \]
\[ \land A → q ∈ P \}. \]
The closure set of an item set is given by reflexive
transitive closure δ*. A correct ELR(0) automa-
ton is given by putting
\[ I_0 = δ^*[\{ S' → q_S : q_S \}], \]
Next(Iq, Z) = δ*τ(Iq, X),
and
\[ \text{Reduce}(I_q, a) = \{ A → q ∈ P \mid \exists [p] ∈ I_q, \alpha ∈ V^*: \]
\[ p ∈ (δ(q, a) ∩ Q_F) \} \]
where
\[ τ(I_q, X) = \{ [p] \mid \exists [q] ∈ I_q, X ∈ V: \]
\[ p ∈ δ(q, X) \}. \]

For example, δ([0]) = [[1]], δ([2]) = [[5]], and δ([6])
= [[9]] for G1. Moreover, I0 = δ*[0] = [[0], [1]]
and Next(I0, a) = δ*τ(I0, a) = [[2], [3], [5]] be-
cause δ[0] = [[1]], τ(I0, a) = [[2], [3]], and δ[2]
= [[5]].

The set I is identified with the set of time sets
recursively, which is the smallest satisfying
\[ I = I_0 \cup \{ δ^*(I_p) \mid \exists I_q ∈ I, X ∈ V: I_p = τ(I_q, X) \}. \]
The behavior and properties of an ELR(0)
automaton can be understood in terms of transi-
tions and paths. A transition (Iq, X) is repre-
sented by \( I_q X \xrightarrow{X} I_p \), where \( I_p = \text{Next}(I_q, X) \). A path is a sequence of states I0, I1, ..., In such that for
some \( X_1, X_2, ..., X_n \),
\[ X_1 I_0 \xrightarrow{X_1} I_1 \xrightarrow{X_2} I_2 \cdots \xrightarrow{X_n} I_{n-1} \xrightarrow{X_n} I_n \]
which is abbreviated \( I_0 \xrightarrow{α} I_n \) where α = \( X_1 X_2 ... X_n \), that is, α accesses \( I_n \).

It is important to note that, whereas in the
LR(0) automaton for a context-free grammar ac-
cessing strings can easily be deduced from state
sequences because each state has a unique entry
symbol [12], in the ELR(0) automaton for an
RRPG accessing strings may not be deduced from
state sequences because each state may have sev-
eral distinct entry symbols. It comes from repeti-
tions and alternations in the description of an
RRPG. The number of states of ELR(0) automa-
ton for an RRPG may be less than that of LR(0)
automaton for corresponding context-free gram-
mar. Therefore, for an RRPG, it is preferred to
use the ELR(0) automaton rather than the LR(0)
automaton.

A relation T on (state, item) pairs is defined
by
\[ (I_q, [q]) T (I_p, [p]) \]
\[ \text{iff } \exists X ∈ V: I_q \xrightarrow{X} I_p \land p ∈ δ(q, X). \]

As shown in [2],
\[ (I_q, [q]) T^*(I_p, [p]) \]
\[ \text{iff } \exists α ∈ V^*: I_q \xrightarrow{α} I_p \land p ∈ δ(q, α). \]

Moreover, T is the union of two different sorts of
relation:
\[ (I_q, [q])_T (I_p, [p]) \]
\[ \text{iff } \exists X ∈ V: I_q \xrightarrow{X} I_p \land [q] ∈ \text{Kernel}(I_q), \]
\[ (I_q, [q])_N (I_p, [p]) \]
\[ \text{iff } \exists [q] ∈ \text{Nonkernel}(I_q). \]

3. Extended LALR(k) parsing of RRPGs

An extended LALR(k) (ELALR(k)) parser for
an RRPG G, ELALR_k(G) = (I, V, P, I0, PT)
where I, V, P, and I0 ∈ I are as in ELR_0(G)
extcept that each item in \( I_q \in I \) is augmented with
the set of lookahead strings of length k, respec-
tively, and PT is a parsing table which consists of
two parts, a parsing action function Action and a
goto function Goto. Action is a mapping from
\( I \times k: \Sigma^* \) to subset of \{ shift I, stack-shift I, re-
duce A → q, accept, error \} where \( k: \Sigma^* \) denotes
the prefix of length k of terminal string [1,12] and
Goto is a mapping from $I \times N$ to \{\text{goto } I, \text{ stack-goto } I\}.

The ELALR(k) parser can be constructed by computing the collection of sets of ELR(0) items and augmenting each item with set of lookahead strings of length $k$. The parsing actions of an ELALR(k) parser and described in terms of relation $\rightarrow$ (read as \textit{moves to}) defined on configurations which consist of state stack and input string. States and vocabularies are stacked as usual. However, a state which is not a lookback state for a reduction by particular production is not stacked [9,11], where lookback state is the state from which the ELR(0) automaton began its search for the handle which is about to reduce. Therefore, a configuration of an ELALR(k) parser is of the form $\langle I_0, \alpha_0, \ldots, I_n, \alpha_n, I_{q_0}, Z \rangle$ where $I_i \in I$, $\alpha_i \in V^*$ for all $i$ such that $0 \leq i \leq n$, $z \in \Sigma^*$, and $I_q \in I$ is the current state.

Basic ELALR(k) parser for RRPG is composed of five kinds of moves as follows:

1. \textit{shift } $I_p$:
   $$\langle I_0, \alpha_0, \ldots, I_n, \alpha_n, I_{q_0}, az \rangle \rightarrow \langle I_0, \alpha_0, I_1, \alpha_1, \ldots, I_n, \alpha_n, I_{q_0}, \alpha Z \rangle$$
   if $I_p = \text{Next}(I_q, a)$ and $\tau_k(I_q, a) \neq \emptyset$,

2. \textit{stack-shift } $I_p$:
   $$\langle I_0, \alpha_0, \ldots, I_n, \alpha_n, I_{q_0}, az \rangle \rightarrow \langle I_0, I_1, I_2, \ldots, I_n, \alpha_n, I_{q_0}, I_{q_0}, az \rangle$$
   if $I_p = \text{Next}(I_q, a)$ and $\tau_k(I_q, a) = \emptyset$,

3. \textit{reduce } $A \rightarrow r$:
   $$\langle I_0, \alpha_0, \ldots, I_n, \alpha_n, I_{q_0}, az \rangle \rightarrow \langle I_0, \alpha_0, I_1, \alpha_1, \ldots, I_n, \alpha_n, I_{q_0} \rangle$$
   if $q \in Q_F$ for $[A \rightarrow r : q] \in L_k$, $\alpha \in LA_k(I_1, [A \rightarrow r : q])$, and $\text{Goto}(I_n, A) = \text{goto } I_p$; or

   $$\langle I_0, \alpha_0, \ldots, I_n, \alpha_n, I_{q_0}, az \rangle \rightarrow \langle I_0, \alpha_0, I_1, \alpha_1, \ldots, I_n, \alpha_n, I_{q_0}, \alpha I_p, az \rangle$$
   if $q \in Q_F$ for $[A \rightarrow r : q] \in L_k$, $\alpha \in LA_k(I_1, [A \rightarrow r : q])$, and $\text{Goto}(I_n, A) = \text{stack-goto } I_p$;

4. \textit{accept}:
   if configuration is of the form $\langle I_0, SI_I, \emptyset \rangle$,

5. \textit{error}:
   otherwise, where $\tau_k(I_q, X)$ is the subset of $\tau(I_q, X)$ such that
   $$\tau_k(I_q, X) = \{ \langle p \rangle | \exists q \in \text{Kernel}(I_q), X \in V : p \in \delta(q, X) \}$$

$LA_k$ is set of lookahead strings of length $k$ for an item in a state, and

$\text{Goto}(I_n, A)$

$$= \left\{ \begin{array}{ll}
\text{goto } I_p & \text{if } I_p = \text{Next}(I_n, A) \text{ and } \tau_k(I_n, A) \neq \emptyset, \\
\text{stack-goto } I_p & \text{if } I_p = \text{Next}(I_n, A) \text{ and } \tau_k(I_n, A) = \emptyset.
\end{array} \right.$$
to each kernel item of a state during the construction of the ELR(0) automaton.

To resolve the stacking conflicts: (i) if stacking conflict of shift action occurs at read time then select stack-shift action, (ii) if there exists stacking conflict transition related to current reduction at reduction time then remove overstacked states from the stack until a lookback state for current reduction appears and perform a reduce action [9]. Also, if stacking conflict of goto occurs then select stack-goto. Additional reduce action is defined as follows:

"reduce \( A \rightarrow r \) to \( LB(I_q, [q]) \)" is the move

\[
(I_0 \alpha_0 \ldots I_n \alpha_n I_{q'}, az)
\]

\[
\vdash (I_0 \alpha_0 I_1 \alpha_1 \ldots I_{n-1} \alpha_{n-1} I_{n'} \alpha_n, az)
\]

if \( q \in Q_F \) for \( [A \rightarrow q] \in I_q \),

\[
k:az \in L A_k(I_q, [A \rightarrow r:q])
\]

and Goto( \( I_m, A \) ) = goto \( I_{n'} \) or

\[
(I_0 \alpha_0 \ldots I_n \alpha_n I_{n'}, az)
\]

\[
\vdash (I_0 \alpha_0 I_1 \alpha_1 \ldots I_{n-1} \alpha_{n-1} I_{n'} \alpha_n, az)
\]

if \( q \in Q_F \) for \( [A \rightarrow r:q] \in I_q \),

\[
k:az \in L A_k(I_q, [A \rightarrow r:q])
\]

and Goto( \( I_m, A \) ) = stack-goto \( I_{n'} \)

where \( I_m \) is the topmost \( I_i \) such that \( I_i \in LB(I_q, [q]) \), and \( \alpha_m \alpha_{m+1} \ldots \alpha_n \) is reduced to \( A \).

The above resolution method can be applied to any ELALR(k) grammar which can be tested during the construction of ELR(0) automaton by the following lemma (the proof of which is analogous to that of the theorem in [9]).

**Lemma.** The ELALR(k) parser for an RRPG is deterministic iff (i) parsing conflicts in inconsistent ELR(0) states can be resolved by using lookahead strings of length \( k \) and (ii) there does not exist a transition \( I_q \overset{X}{\rightarrow} I_p \) such that there exist distinct two items \( [q],[q'] \in I_q \), \( [r],[r'] \in I_r \), and

\[
(I_q, [q])T(I_p, [p])
\]

\[
and \quad (I_q, [q'])T(I_p, [p])
\]

where

\[
(I_r, [r]_k T^*(I_q', [q]))
\]

\[
and \quad (I_r, [r']_k T^*(I_q', [q'])) \quad \square
\]

If \( [r] \in \text{Nonkernel}(I_r) \) in the lemma, lookback state for a reduction is unique. And then the handle to be reduced is uniquely determined. Therefore, the ELALR(k) parser for an RRPG is deterministic iff parsing conflicts in inconsistent ELR(0) states can be resolved by using lookahead strings of length \( k \) and lookback states.

**Algorithm E.** Generation of ELALR parser from ELR(0) automaton for RRPG \( G \) with underlying item sets of \( Q \).

for \( I_q \in I \)

for \( X \in \mathcal{V} \) where \( \exists I_p \in I : I_p = \text{Next}(I_q, X) \)

Action[ \( I_q, X \) ] := shift \( I_p \)

if \( \tau_k(I_q, X) = \text{Kernel}(I_p) \land X \in \Sigma \);

Action[ \( I_q, X \) ] := stack-shift \( I_p \)

if \( \tau_k(I_q, X) \neq \text{Kernel}(I_p) \land X \in \Sigma \);

Action[ \( I_q, X \) ] := goto \( I_p \)

if \( \tau_k(I_q, X) = \text{Kernel}(I_p) \land X \in N \);

Action[ \( I_q, X \) ] := stack-goto \( I_p \)

if \( \tau_k(I_q, X) \neq \text{Kernel}(I_p) \land X \in N \);

if \( \tau_k(I_q, X) \neq \emptyset \) and

\[
\tau_k(I_q, X) \neq \text{Kernel}(I_p)
\]

then for \( [p] \in \tau_k(I_q, X) \)

mark \( [p] \) and all \( [r] \) such that

\[
(I_p, [p]) T^*(I_r, [r]);
\]

for \( I_p \in I \) where \( \exists I_p \in Q_F : [A \rightarrow q;p] \in I_p \)

for \( z \in L A_k(I_p, A \rightarrow q) \)

Action[ \( I_p, z \) ] := reduce \( A \rightarrow q \)

if \( [p] \) is not marked;

Action[ \( I_p, z \) ] := reduce \( A \rightarrow q \)

if \( [p] \) is marked;

Action[ \( I_p, \$^k \) ] := accept

if \( [S' \rightarrow q;:p] \in I_p \);

\( \mathcal{L} \)

**Note.** Algorithm \( E \) uses only kernel items. However, when the final right part state of some reduce item is also initial right part state, that is, a production generates \( \epsilon \), the reduce item is nonkernel. Such item should be used to generate ELALR parser in Algorithm \( E \) even though it is nonkernel item.
Example. The ELALR(1) parser for G1 is shown in Fig. 2 using pictorial representation for parsing table. "|" separates kernel items and nonkernel items of each state. For example, Kernel(I2) = {[2],[3]} and Nonkernel(I2) = {[5]}. The edges are labeled by shift, goto, stack-shift, and stack-goto on X, respectively. A state where a reduction is possible is annotated by "#" with production number and lookahead set. Also, #1[5]: LB = {I0} indicates that the action is reduce #1 to lookback state set {I0} with lookahead set {5}. Moreover, \( \tau_K \) and relation \( T \) only on (state, kernel item) pairs are as follows:

\[
\begin{align*}
\tau_K(I2, b) &= \{[3]\}, \\
\tau_K(I2, c) &= \tau_K(I2, A) = \tau_K(I5, c) = \{[4]\}, \\
\tau_K(I3, b) &= \tau_K(I5, b) = \{[3]\}, \\
\tau_K(I3, A) &= \{[4]\}, \\
\tau_K(I5, c) &= \{[8]\}, \\
\tau_K(I6, c) &= \{[8]\}, \\
\tau_K(I8, b) &= \{[11]\}, \\
(I2, [2])T(I3, [3]), & \quad (I2, [3])T(I3, [3]), \\
(I2, [4])T(I4, [4]), & \quad (I5, [3])T(I6, [4]), \\
(I3, [3])T(I5, [3]), & \quad (I3, [6])T(I6, [8]), \\
(I3, [6])T(I6, [7]), & \quad (I5, [3])T(I6, [3]), \\
(I5, [4])T(I7, [8]), & \quad (I8, [10])T(I9, [11]).
\end{align*}
\]

Action[0, S] = stack-goto I1 because \( \tau_K(I0, S) = \emptyset \). Kernel(I1) = {[12]}, and \( S \in N \). Action[I0, a] = stack-shift I2 because \( \tau_K(I0, a) = \emptyset \). Kernel(I2) = {[2],[3]}, and \( a \in \Sigma \). Action[I2, c] = shift I4 because \( \tau_K(I2, c) = \{[4]\} \), Kernel(I4) = {[4]}, and \( c \in \Sigma \). Action[I2, A] = goto I4 because \( \tau_K(I2, A) = \{[4]\} \), Kernel(I4) = {[4]}, and \( A \in N \).

There are two distinct accessing strings on the sequence of state I2-I3-I4 such as ac and aA. Stacking conflict occurs at the transition \( I2 \xrightarrow{b} I3 \) because \( \tau_K(I2, b) = \{[3]\} \neq \emptyset \) and Kernel(I2) = {[3],[6]}. Also, stacking conflict occurs at the transition \( I3 \xrightarrow{c} I8 \). Therefore, \{3\} \( \in I3 \) is marked as \([3]^*\) and this marking is transferred to \{3\} \( \in I8 \) because \( (I3, [3])T(I3, [3]), (I3, [3])T(I3, [3]), (I3, [3])T(I3, [3]), (I3, [3])T(I3, [3]) \). To resolve stacking conflicts, Action[I2, b] = stack-shift I5, Action[I3, c] = stack-shift I8. Although \( (I2, [2])T(I3, [3]) \) and \( (I2, [3])T(I3, [3]) \), the handle to be reduced by production \( S \rightarrow 1 \) in state I4 can be uniquely determined by using lookahead state \( I0 \) because \( (I0, [1])_N T(I2, [2]) \) and \( (I0, [1])_N T(I2, [3]) \). Therefore, the action for lookahead ($) at I4 and I8 is "remove the states from stack until I0 appears, then reduce by the rule #1" because LB = {I0}.

If the input string is "abcbe$" then the parsing will proceed as

\[
\cdots (I0aI2bI3cI5, be$) \xrightarrow{\text{shift}} (I0aI2bI3cI5, c$) \xrightarrow{\text{reduce}#1\rightarrow cb} (I0aI2bI6, c$)
\]

Fig. 2. ELALR(1) parser for G1.
\[ \text{\textbackslash lift}_{\text{shift}}(I_0 a I_2 b B c I_7, $) \]
\[ \text{\textbackslash lift}_{\text{reduce}#2(A \rightarrow b B c)}(I_0 a A I_4, $) \ldots \]

and if "abbc$" is given then

\[ \cdots (I_0 a I_2 b I_3, b b c$) \text{\textbackslash lift}_{\text{shift}}(I_0 a I_2 b b I_5, b c$) \]
\[ \text{\textbackslash lift}_{\text{shift}}(I_0 a I_2 b b b I_5, c$) \text{\textbackslash lift}_{\text{shift}}(I_0 a I_2 b b b c I_4, $) \]
\[ \text{\textbackslash lift}_{\text{reduce}#1 to \{I_0\}(S \rightarrow abbc)}(I_0 S I_1, $) \ldots \]

4. Conclusion

We presented an improved method in generation of efficient ELALR(k) parsers for RRPGs, in which only kernel items of the ELR(0) states are used. It is likely to incur large space overheads in explicitly keeping relations between (state, item) pairs [2,9]. It can be reduced by using only kernel items on building the parsers with the new operator $\tau_K$ which is used to test and resolve stacking conflicts.

Moreover, in our method, right parts of productions of RRPGs are nondeterministic finite automata. During the construction of ELR(0) automaton, fewer number of items may be required because nondeterministic finite automata can be exponentially more succinct than deterministic finite automata as shown in [12].

References